THEOREM 5.8 *Let $Y_1, \ldots, Y_n$ and $X_1, \ldots, X_m$ be random variables with $E(Y_i) = \mu_i$ and $E(X_j) = \xi_j$. Define*

$$\boxed{U_1 = \sum_{i=1}^{n} a_i Y_i} \qquad U_2 = \sum_{j=1}^{m} b_j X_j$$

for constants $a_1, \ldots, a_n, b_1, \ldots, b_m$. Then the following hold:

(a) $E(U_1) = \sum_{i=1}^{n} a_i \mu_i$.

(b) $V(U_1) = \sum_{i=1}^{n} a_i^2 V(Y_i) + \boxed{2 \sum\sum_{i<j} a_i a_j \operatorname{Cov}(Y_i, Y_j)}$, *where the double sum is over all pairs (i, j) with $i < j$.*

(c) $\operatorname{Cov}(U_1, U_2) = \sum_{i=1}^{n} \sum_{j=1}^{m} a_i b_j \operatorname{Cov}(Y_i, X_j)$.