The pdf of the central *t*-distribution on *df* degrees of freedom

$$f_t(t|df) = \frac{\Gamma\left(\frac{df+1}{2}\right)}{\sqrt{df\pi}\,\Gamma\left(\frac{df}{2}\right)}\left(1 - \frac{1}{df}t^2\right)^{-\frac{df+1}{2}}$$

Typical (i.e., historical) use: we have $\hat{\mu}$ (often it is a simple mean) with estimated standard error, $\hat{\text{se}}$. For inference with

$$t = \frac{\hat{\mu}}{\hat{\text{se}}}$$

and compute a *P*-value (say 2-sided) as

$$P = 2\int_{|t|}^{\infty} f_t(x|df)dx.$$

A simpler, likelihood inferential proceedure exists based recognizing a pivitol quantity as

$$t = \frac{\hat{\mu}-\mu}{\hat{\text{se}}}$$ taken as having a central *t*-distrubution.

A likelihood in μ, standardized to a maximum value of 1 is

$$L(\mu) = \left(1 - \frac{1}{df}\left(\frac{\hat{\mu}-\mu}{\hat{\text{se}}}\right)^2\right)^{-\frac{df+1}{2}}.$$

The best supported estimate is $\hat{\mu}$, and evidence about other values of μ is then relative to this MLE. Use $L(\mu)$ to compute evidence as e:1 odds. I prefer to have “e” ≥ 1 hence flip $L(\mu)$ over and use as odds ***against*** another μ

$$\left(1 - \frac{1}{df}\left(\frac{\hat{\mu}-\mu}{\hat{\text{se}}}\right)^2\right)^{\frac{df+1}{2}} :1$$

For example, if df $= 7$ and we consider $\mu = 0$, for which case say we find $t = 2.36462$, then odds against $\mu = 0$, relative to $\hat{\mu}$ are easily computed as

$(1 - \frac{1}{7}(2.36462)^2)^4$:1

10.47 :1

Note 1. Take this is one-sided as we pay attention to the sign of $\hat{\mu}$.
Note 2. For this case, 2-sided $P = 0.05$.

For the “normal distribution” case it is $\exp(z^2/2)$:1 against $\mu = 0$.

Strenngth of inference is expressed as odds, not as probability; easy to compute, and likelihood-based.

What is wrong with this - if anything? Why don't we (statistics) use it for inference **given the data?** Probability theory as already developed is still to be used in planning stages, like sample size assessment. However, this way our thinking and methods are different before we have the data vs. after we have the data. (There exists much more thoughts and literature about this, but I am keeping it short here.)