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**Comparison of LHS Design**

**With**

**Half Fraction of Two Factorial Design**

**and Various Fractions of Three Factorial**

**Design In Identifying Non-Linearities**

**And**

**In Avoiding Confounding Effects**

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### Summary & Conclusion

The classical experiments based on the fractional factorial designs are typically used to study the main effects along with selected interaction terms. The limitations of these experimental design methods to effectively capture all interaction effects are well known. This paper demonstrates the ability of the Latin Hypercube Sampling (LHS) method to overcome the problems associated with the classic fractional factorial experimental design. The study is conducted using the four explanatory variables. The LHS design is implemented using only eight points. The half fraction of two factorial experimental designs is used for comparison purposes. In addition to the four main effects, the analysis also includes four second order and eleven interaction terms. The paper also looks at the various fractions of complete three factorial design for the comparative purposes. In the three factorial design, the number of points vary from maximum of 81 points to as little as nine points.

Based on the analysis of the outputs of these experiments, it is clear that the LHS experimental design does not experience "confounding" effects traditionally associated with the classical fractional factorial experimental designs. The eight point LHS design is most reliable compared to fractional three factorial design consisting of nine or more points.

Functional Form of the Regression Model Used

$$\begin{aligned}
 Y = & 1 x_1 + 2 x_2 + 3 x_3 + 4 x_4 \\
 & + 5 x_1 x_1 + 6 x_2 x_2 + 7 x_3 x_3 + 8 x_4 x_4 \\
 & + 9 x_1 x_2 + 10 x_1 x_3 + 11 x_1 x_4 + 12 x_2 x_3 \\
 & + 13 x_2 x_4 + 14 x_3 x_4 + 15 x_1 x_2 x_3 \\
 & + 16 x_1 x_2 x_4 + 17 x_1 x_3 x_4 + 18 x_2 x_3 x_4 \\
 & + 19 x_1 x_2 x_3 x_4
 \end{aligned}$$

inputs:

## Half Fraction of Two Factorial Design

| x1 | x2 | x3 | x4 |
|----|----|----|----|
| 1  | -1 | -1 | -1 |
| -1 | 1  | -1 | -1 |
| -1 | -1 | 1  | -1 |
| -1 | -1 | -1 | 1  |
| 1  | 1  | 1  | -1 |
| 1  | 1  | -1 | 1  |
| -1 | 1  | 1  | 1  |
| 1  | -1 | 1  | 1  |

## Latin Hypercube Sample Design

Sample 1

## Induced Rank Correlation Matrix:

| x1     | x2     | x3     | x4     |        |        |        |        |
|--------|--------|--------|--------|--------|--------|--------|--------|
| -.5556 | -.5556 | .3333  | -.3333 | 1.0000 | .0238  | .1429  | .0714  |
| -.1111 | -.7778 | -.3333 | .5556  | .0238  | 1.0000 | .0238  | .0476  |
| .7778  | .1111  | -.7778 | -.1111 | .1429  | .0238  | 1.0000 | -.0238 |
| -.3333 | .7778  | -.1111 | .7778  | .0714  | .0476  | -.0238 | 1.0000 |
| .3333  | .5556  | .7778  | .1111  |        |        |        |        |
| .5556  | -.1111 | .5556  | -.5556 |        |        |        |        |
| -.7778 | .3333  | -.5556 | -.7778 |        |        |        |        |
| .1111  | -.3333 | .1111  | .3333  |        |        |        |        |

Sample 2

| x1     | x2     | x3     | x4     |
|--------|--------|--------|--------|
| -.5556 | -.5556 | -.3333 | .3333  |
| -.1111 | -.7778 | .5556  | -.1111 |
| .7778  | .1111  | -.1111 | .1111  |
| -.3333 | .7778  | .7778  | .7778  |
| .3333  | .5556  | .1111  | -.7778 |
| .5556  | -.1111 | -.5556 | .5556  |
| -.7778 | .3333  | -.7778 | -.3333 |
| .1111  | -.3333 | .3333  | -.5556 |

Induced Rank Correlation Matrix:

|        |        |        |        |
|--------|--------|--------|--------|
| 1.0000 | .0238  | .0714  | -.0476 |
| .0238  | 1.0000 | .0476  | .0476  |
| .0714  | .0476  | 1.0000 | .0476  |
| -.0476 | .0476  | .0476  | 1.0000 |

Half Fraction of Two Factorial Design

|               |       |       |       |       |       |       |       |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| actual Model: | x1    | x2    | x3    | x4    | x11   | x22   | x33   |
| fitted Coefs: | 1.00  | 2.00  | 3.00  | 4.00  | 6.00  | 6.00  | 6.00  |
| actual Model: | x2    | x3    | x4    | x11   | x22   | x33   | x44   |
| fitted Coefs: | 2.00  | 3.00  | 4.00  | 6.50  | 6.50  | 6.50  | 6.50  |
| actual Model: | x3    | x4    | x11   | x22   | x33   | x44   | x12   |
| fitted Coefs: | 3.00  | 4.00  | 6.50  | 6.50  | 6.50  | 6.50  | 9.00  |
| actual Model: | x4    | x11   | x22   | x33   | x44   | x12   | x13   |
| fitted Coefs: | 4.00  | 6.50  | 6.50  | 6.50  | 6.50  | 9.00  | 10.00 |
| actual Model: | x11   | x22   | x33   | x44   | x12   | x13   | x14   |
| fitted Coefs: | 6.50  | 6.50  | 6.50  | 6.50  | 9.00  | 10.00 | 11.00 |
| actual Model: | x22   | x33   | x44   | x12   | x13   | x14   | x23   |
| fitted Coefs: | 7.00  | 7.00  | 7.00  | 9.00  | 10.00 | -.50  | .50   |
| actual Model: | x33   | x44   | x12   | x13   | x14   | x23   | x24   |
| fitted Coefs: | 7.50  | 7.50  | 9.00  | -1.50 | -.50  | .50   | 1.50  |
| actual Model: | x44   | x12   | x13   | x14   | x23   | x24   | x34   |
| fitted Coefs: | 8.00  | -2.50 | -1.50 | -.50  | .50   | 1.50  | 2.50  |
| actual Model: | x12   | x13   | x14   | x23   | x24   | x34   | x123  |
| fitted Coefs: | -2.50 | -1.50 | -.50  | .50   | 1.50  | 2.50  | 15.00 |
| actual Model: | x13   | x14   | x23   | x24   | x34   | x123  | x124  |
| fitted Coefs: | -1.50 | -.50  | .50   | 1.50  | 14.00 | 15.00 | 16.00 |
| actual Model: | x14   | x23   | x24   | x34   | x123  | x124  | x134  |
| fitted Coefs: | -.50  | .50   | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 |
| actual Model: | x23   | x24   | x34   | x123  | x124  | x134  | x234  |
| fitted Coefs: | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24*  | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24*  | x34*  | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24*  | x34*  | x123* | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24*  | x34*  | x123* | x124* | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | 17.00 | 18.00 | 19.00 |
| actual Model: | x24*  | x34*  | x123* | x124* | x134* | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | 18.00 | 19.00 |
| actual Model: | x24*  | x34*  | x123* | x124* | x134* | x234* | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | .00   | 19.00 |

\* - model coefficients were set to zero

Latin Hypercube Design

|               |       |       |       |       |       |       |       |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| actual Model: | x1    | x2    | x3    | x4    | x11   | x22   | x33   |
| fitted Coefs: | 1.00  | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  |
| actual Model: | x2    | x3    | x4    | x11   | x22   | x33   | x44   |
| fitted Coefs: | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  |
| actual Model: | x3    | x4    | x11   | x22   | x33   | x44   | x12   |
| fitted Coefs: | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  |
| actual Model: | x4    | x11   | x22   | x33   | x44   | x12   | x13   |
| fitted Coefs: | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 |
| actual Model: | x11   | x22   | x33   | x44   | x12   | x13   | x14   |
| fitted Coefs: | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 |
| actual Model: | x22   | x33   | x44   | x12   | x13   | x14   | x23   |
| fitted Coefs: | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 |
| actual Model: | x33   | x44   | x12   | x13   | x14   | x23   | x24   |
| fitted Coefs: | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 |
| actual Model: | x44   | x12   | x13   | x14   | x23   | x24   | x34   |
| fitted Coefs: | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 |
| actual Model: | x12   | x13   | x14   | x23   | x24   | x34   | x123  |
| fitted Coefs: | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 |
| actual Model: | x13   | x14   | x23   | x24   | x34   | x123  | x124  |
| fitted Coefs: | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 |
| actual Model: | x14   | x23   | x24   | x34   | x123  | x124  | x134  |
| fitted Coefs: | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 |
| actual Model: | x23   | x24   | x34   | x123  | x124  | x134  | x234  |
| fitted Coefs: | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24*  | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24*  | x34*  | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24*  | x34*  | x123* | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24*  | x34*  | x123* | x124* | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | 17.00 | 18.00 | 19.00 |
| actual Model: | x24*  | x34*  | x123* | x124* | x134* | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | 18.00 | 19.00 |
| actual Model: | x24*  | x34*  | x123* | x124* | x134* | x234* | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | .00   | 19.00 |

\* - Model coefficients were set to zero

81 (All Points)

|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
|    |    |    |    | 0  | 0  | 0  | 0  |
|    |    |    |    | +1 | 0  | 0  | 0  |
| -1 | -1 | -1 | -1 | -1 | +1 | 0  | 0  |
| 0  | -1 | -1 | -1 | 0  | +1 | 0  | 0  |
| +1 | -1 | -1 | -1 | +1 | +1 | 0  | 0  |
| -1 | 0  | -1 | -1 | -1 | -1 | +1 | 0  |
| 0  | 0  | -1 | -1 | 0  | -1 | +1 | 0  |
| +1 | 0  | -1 | -1 | +1 | -1 | +1 | 0  |
| -1 | +1 | -1 | -1 | -1 | 0  | +1 | 0  |
| 0  | +1 | -1 | -1 | 0  | 0  | +1 | 0  |
| +1 | +1 | -1 | -1 | +1 | 0  | +1 | 0  |
| -1 | -1 | 0  | -1 | -1 | +1 | +1 | 0  |
| 0  | -1 | 0  | -1 | 0  | +1 | +1 | 0  |
| +1 | -1 | 0  | -1 | +1 | +1 | +1 | 0  |
| -1 | 0  | 0  | -1 | -1 | -1 | -1 | +1 |
| 0  | 0  | 0  | -1 | 0  | -1 | -1 | +1 |
| +1 | 0  | 0  | -1 | +1 | -1 | -1 | +1 |
| -1 | +1 | 0  | -1 | -1 | 0  | -1 | +1 |
| 0  | +1 | 0  | -1 | 0  | 0  | -1 | +1 |
| +1 | +1 | 0  | -1 | +1 | 0  | -1 | +1 |
| -1 | -1 | +1 | -1 | -1 | +1 | -1 | +1 |
| 0  | -1 | +1 | -1 | 0  | +1 | -1 | +1 |
| +1 | -1 | +1 | -1 | +1 | +1 | -1 | +1 |
| -1 | 0  | +1 | -1 | -1 | -1 | 0  | +1 |
| 0  | 0  | +1 | -1 | 0  | -1 | 0  | +1 |
| +1 | 0  | +1 | -1 | +1 | -1 | 0  | +1 |
| -1 | +1 | +1 | -1 | -1 | 0  | 0  | +1 |
| 0  | +1 | +1 | -1 | 0  | 0  | 0  | +1 |
| +1 | +1 | +1 | -1 | +1 | 0  | 0  | +1 |
| -1 | -1 | -1 | 0  | -1 | +1 | 0  | +1 |
| 0  | -1 | -1 | 0  | 0  | +1 | 0  | +1 |
| +1 | -1 | -1 | 0  | +1 | +1 | 0  | +1 |
| -1 | 0  | -1 | 0  | -1 | -1 | +1 | +1 |
| 0  | 0  | -1 | 0  | 0  | -1 | +1 | +1 |
| +1 | 0  | -1 | 0  | +1 | -1 | +1 | +1 |
| -1 | +1 | -1 | 0  | -1 | 0  | +1 | +1 |
| 0  | +1 | -1 | 0  | 0  | 0  | +1 | +1 |
| +1 | +1 | -1 | 0  | +1 | 0  | +1 | +1 |
| -1 | -1 | 0  | 0  | -1 | +1 | +1 | +1 |
| 0  | -1 | 0  | 0  | 0  | +1 | +1 | +1 |
| +1 | -1 | 0  | 0  | +1 | +1 | +1 | +1 |
| -1 | 0  | 0  | 0  | +1 | +1 | +1 | +1 |

|               |       |       |       |       |       |       |       |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| actual Model: | x1    | x2    | x3    | x4    | x11   | x22   | x33   |
| fitted Coefs: | 1.00  | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  |
| actual Model: | x2    | x3    | x4    | x11   | x22   | x33   | x44   |
| fitted Coefs: | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  |
| actual Model: | x3    | x4    | x11   | x22   | x33   | x44   | x12   |
| fitted Coefs: | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  |
| actual Model: | x4    | x11   | x22   | x33   | x44   | x12   | x13   |
| fitted Coefs: | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 |
| actual Model: | x11   | x22   | x33   | x44   | x12   | x13   | x14   |
| fitted Coefs: | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 |
| actual Model: | x22   | x33   | x44   | x12   | x13   | x14   | x23   |
| fitted Coefs: | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 |
| actual Model: | x33   | x44   | x12   | x13   | x14   | x23   | x24   |
| fitted Coefs: | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 |
| actual Model: | x44   | x12   | x13   | x14   | x23   | x24   | x34   |
| fitted Coefs: | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 |
| actual Model: | x12   | x13   | x14   | x23   | x24   | x34   | x123  |
| fitted Coefs: | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 |
| actual Model: | x13   | x14   | x23   | x24   | x34   | x123  | x124  |
| fitted Coefs: | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 |
| actual Model: | x14   | x23   | x24   | x34   | x123  | x124  | x134  |
| fitted Coefs: | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 |
| actual Model: | x23   | x24   | x34   | x123  | x124  | x134  | x234  |
| fitted Coefs: | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | .00   | 19.00 |

41 Points (first half of the total population)

|    |    |    |    |
|----|----|----|----|
| -1 | -1 | -1 | -1 |
| 0  | -1 | -1 | -1 |
| +1 | -1 | -1 | -1 |
| -1 | 0  | -1 | -1 |
| 0  | 0  | -1 | -1 |
| +1 | 0  | -1 | -1 |
| -1 | +1 | -1 | -1 |
| 0  | +1 | -1 | -1 |
| +1 | +1 | -1 | -1 |
| -1 | -1 | 0  | -1 |
| 0  | -1 | 0  | -1 |
| +1 | -1 | 0  | -1 |
| -1 | 0  | 0  | -1 |
| 0  | 0  | 0  | -1 |
| +1 | 0  | 0  | -1 |
| -1 | +1 | 0  | -1 |
| 0  | +1 | 0  | -1 |
| +1 | +1 | 0  | -1 |
| -1 | -1 | +1 | -1 |
| 0  | -1 | +1 | -1 |
| +1 | -1 | +1 | -1 |
| -1 | 0  | +1 | -1 |
| 0  | 0  | +1 | -1 |
| +1 | 0  | +1 | -1 |
| -1 | +1 | +1 | -1 |
| 0  | +1 | +1 | -1 |
| +1 | +1 | +1 | -1 |
| -1 | -1 | -1 | 0  |
| 0  | -1 | -1 | 0  |
| +1 | -1 | -1 | 0  |
| -1 | 0  | -1 | 0  |
| 0  | 0  | -1 | 0  |
| +1 | 0  | -1 | 0  |
| -1 | +1 | -1 | 0  |
| 0  | +1 | -1 | 0  |
| +1 | +1 | -1 | 0  |
| -1 | -1 | 0  | 0  |
| 0  | -1 | 0  | 0  |
| +1 | -1 | 0  | 0  |
| -1 | 0  | 0  | 0  |
| 0  | 0  | 0  | 0  |

|               |              |              |              |       |             |             |             |
|---------------|--------------|--------------|--------------|-------|-------------|-------------|-------------|
| actual Model: | x1           | x2           | x3           | x4    | x11         | x22         | x33         |
| fitted Coefs: | 1.00         | 2.00         | 3.00         | 4.00  | 5.00        | 6.00        | 7.00        |
| actual Model: | x2           | x3           | x4           | x11   | x22         | x33         | x44         |
| fitted Coefs: | 2.00         | 3.00         | <u>-2.00</u> | 5.00  | 6.00        | 7.00        | <u>2.00</u> |
| actual Model: | x3           | x4           | x11          | x22   | x33         | x44         | x12         |
| fitted Coefs: | 3.00         | <u>-2.00</u> | 5.00         | 6.00  | 7.00        | <u>2.00</u> | 9.00        |
| actual Model: | x4           | x11          | x22          | x33   | x44         | x12         | x13         |
| fitted Coefs: | <u>-2.00</u> | 5.00         | 6.00         | 7.00  | <u>2.00</u> | 9.00        | 10.00       |
| actual Model: | x11          | x22          | x33          | x44   | x12         | x13         | x14         |
| fitted Coefs: | 5.00         | 6.00         | 7.00         | 8.00  | 9.00        | 10.00       | 11.00       |
| actual Model: | x22          | x33          | x44          | x12   | x13         | x14         | x23         |
| fitted Coefs: | 6.00         | 7.00         | 8.00         | 9.00  | 10.00       | 11.00       | 12.00       |
| actual Model: | x33          | x44          | x12          | x13   | x14         | x23         | x24         |
| fitted Coefs: | 7.00         | 8.00         | 9.00         | 10.00 | 11.00       | 12.00       | 13.00       |
| actual Model: | x44          | x12          | x13          | x14   | x23         | x24         | x34         |
| fitted Coefs: | 8.00         | 9.00         | 10.00        | 11.00 | 12.00       | 13.00       | 14.00       |
| actual Model: | x12          | x13          | x14          | x23   | x24         | x34         | x123        |
| fitted Coefs: | 9.00         | 10.00        | 11.00        | 12.00 | 13.00       | 14.00       | 15.00       |
| actual Model: | x13          | x14          | x23          | x24   | x34         | x123        | x124        |
| fitted Coefs: | 10.00        | 11.00        | 12.00        | 13.00 | 14.00       | 15.00       | 16.00       |
| actual Model: | x14          | x23          | x24          | x34   | x123        | x124        | x134        |
| fitted Coefs: | 11.00        | 12.00        | 13.00        | 14.00 | 15.00       | 16.00       | 17.00       |
| actual Model: | x23          | x24          | x34          | x123  | x124        | x134        | x234        |
| fitted Coefs: | 12.00        | 13.00        | 14.00        | 15.00 | 16.00       | 17.00       | 18.00       |
| actual Model: | x24          | x34          | x123         | x124  | x134        | x234        | x1234       |
| fitted Coefs: | 13.00        | 14.00        | 15.00        | 16.00 | 17.00       | 18.00       | 19.00       |
| actual Model: | x24          | x34          | x123         | x124  | x134        | x234        | x1234       |
| fitted Coefs: | .00          | 14.00        | 15.00        | 16.00 | 17.00       | 18.00       | 19.00       |
| actual Model: | x24          | x34          | x123         | x124  | x134        | x234        | x1234       |
| fitted Coefs: | .00          | .00          | 15.00        | 16.00 | 17.00       | 18.00       | 19.00       |
| actual Model: | x24          | x34          | x123         | x124  | x134        | x234        | x1234       |
| fitted Coefs: | .00          | .00          | .00          | 16.00 | 17.00       | 18.00       | 19.00       |
| actual Model: | x24          | x34          | x123         | x124  | x134        | x234        | x1234       |
| fitted Coefs: | .00          | .00          | .00          | .00   | 17.00       | 18.00       | 19.00       |
| actual Model: | x24          | x34          | x123         | x124  | x134        | x234        | x1234       |
| fitted Coefs: | .00          | .00          | .00          | .00   | .00         | 18.00       | 19.00       |
| actual Model: | x24          | x34          | x123         | x124  | x134        | x234        | x1234       |
| fitted Coefs: | .00          | .00          | .00          | .00   | .00         | .00         | 19.00       |

41 Points (Every other point in the population)

|    |    |    |    |
|----|----|----|----|
| -1 | -1 | -1 | -1 |
| +1 | -1 | -1 | -1 |
| 0  | 0  | -1 | -1 |
| -1 | +1 | -1 | -1 |
| +1 | +1 | -1 | -1 |
| 0  | -1 | 0  | -1 |
| -1 | 0  | 0  | -1 |
| +1 | 0  | 0  | -1 |
| 0  | +1 | 0  | -1 |
| -1 | -1 | +1 | -1 |
| +1 | -1 | +1 | -1 |
| 0  | 0  | +1 | -1 |
| -1 | +1 | +1 | -1 |
| +1 | +1 | +1 | -1 |
| 0  | -1 | -1 | 0  |
| -1 | 0  | -1 | 0  |
| +1 | 0  | -1 | 0  |
| 0  | +1 | -1 | 0  |
| -1 | -1 | 0  | 0  |
| +1 | -1 | 0  | 0  |
| 0  | 0  | 0  | 0  |
| -1 | +1 | 0  | 0  |
| +1 | +1 | 0  | 0  |
| 0  | -1 | +1 | 0  |
| -1 | 0  | +1 | 0  |
| +1 | 0  | +1 | 0  |
| 0  | +1 | +1 | 0  |
| -1 | -1 | -1 | +1 |
| +1 | -1 | -1 | +1 |
| 0  | 0  | -1 | +1 |
| -1 | +1 | -1 | +1 |
| +1 | +1 | -1 | +1 |
| 0  | -1 | 0  | +1 |
| -1 | 0  | 0  | +1 |
| +1 | 0  | 0  | +1 |
| 0  | +1 | 0  | +1 |
| -1 | -1 | +1 | +1 |
| +1 | -1 | +1 | +1 |
| 0  | 0  | +1 | +1 |
| -1 | +1 | +1 | +1 |
| +1 | +1 | +1 | +1 |

|               |       |       |       |       |       |       |       |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| actual Model: | x1    | x2    | x3    | x4    | x11   | x22   | x33   |
| fitted Coefs: | 1.00  | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  |
| actual Model: | x2    | x3    | x4    | x11   | x22   | x33   | x44   |
| fitted Coefs: | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  |
| actual Model: | x3    | x4    | x11   | x22   | x33   | x44   | x12   |
| fitted Coefs: | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  |
| actual Model: | x4    | x11   | x22   | x33   | x44   | x12   | x13   |
| fitted Coefs: | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 |
| actual Model: | x11   | x22   | x33   | x44   | x12   | x13   | x14   |
| fitted Coefs: | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 |
| actual Model: | x22   | x33   | x44   | x12   | x13   | x14   | x23   |
| fitted Coefs: | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 |
| actual Model: | x33   | x44   | x12   | x13   | x14   | x23   | x24   |
| fitted Coefs: | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 |
| actual Model: | x44   | x12   | x13   | x14   | x23   | x24   | x34   |
| fitted Coefs: | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 |
| actual Model: | x12   | x13   | x14   | x23   | x24   | x34   | x123  |
| fitted Coefs: | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 |
| actual Model: | x13   | x14   | x23   | x24   | x34   | x123  | x124  |
| fitted Coefs: | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 |
| actual Model: | x14   | x23   | x24   | x34   | x123  | x124  | x134  |
| fitted Coefs: | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 |
| actual Model: | x23   | x24   | x34   | x123  | x124  | x134  | x234  |
| fitted Coefs: | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | .00   | 19.00 |

21 Points ( Every fourth point in the Population)

|    |    |    |    |
|----|----|----|----|
| -1 | -1 | -1 | -1 |
| 0  | 0  | -1 | -1 |
| +1 | +1 | -1 | -1 |
| -1 | 0  | 0  | -1 |
| 0  | +1 | 0  | -1 |
| +1 | -1 | +1 | -1 |
| -1 | +1 | +1 | -1 |
| 0  | -1 | -1 | 0  |
| +1 | 0  | -1 | 0  |
| -1 | -1 | 0  | 0  |
| 0  | 0  | 0  | 0  |
| +1 | +1 | 0  | 0  |
| -1 | 0  | +1 | 0  |
| 0  | +1 | +1 | 0  |
| +1 | -1 | -1 | +1 |
| -1 | +1 | -1 | +1 |
| 0  | -1 | 0  | +1 |
| +1 | 0  | 0  | +1 |
| -1 | -1 | +1 | +1 |
| 0  | 0  | +1 | +1 |
| +1 | +1 | +1 | +1 |

|               |       |       |       |       |       |       |       |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| actual Model: | x1    | x2    | x3    | x4    | x11   | x22   | x33   |
| fitted Coefs: | 1.00  | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  |
| actual Model: | x2    | x3    | x4    | x11   | x22   | x33   | x44   |
| fitted Coefs: | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  |
| actual Model: | x3    | x4    | x11   | x22   | x33   | x44   | x12   |
| fitted Coefs: | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  |
| actual Model: | x4    | x11   | x22   | x33   | x44   | x12   | x13   |
| fitted Coefs: | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 |
| actual Model: | x11   | x22   | x33   | x44   | x12   | x13   | x14   |
| fitted Coefs: | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 |
| actual Model: | x22   | x33   | x44   | x12   | x13   | x14   | x23   |
| fitted Coefs: | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 |
| actual Model: | x33   | x44   | x12   | x13   | x14   | x23   | x24   |
| fitted Coefs: | 7.83  | 7.17  | 9.00  | 10.83 | 11.83 | 11.17 | 12.17 |
| actual Model: | x44   | x12   | x13   | x14   | x23   | x24   | x34   |
| fitted Coefs: | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 |
| actual Model: | x12   | x13   | x14   | x23   | x24   | x34   | x123  |
| fitted Coefs: | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 |
| actual Model: | x13   | x14   | x23   | x24   | x34   | x123  | x124  |
| fitted Coefs: | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 |
| actual Model: | x14   | x23   | x24   | x34   | x123  | x124  | x134  |
| fitted Coefs: | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 |
| actual Model: | x23   | x24   | x34   | x123  | x124  | x134  | x234  |
| fitted Coefs: | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | .00   | 19.00 |

17 Points ( Every fifth point in the population)

```

-1 -1 -1 -1
+1  0 -1 -1
  0 -1  0 -1
-1 +1  0 -1
+1 -1 +1 -1
  0 +1 +1 -1
-1  0 -1  0
+1 +1 -1  0
  0  0  0  0
-1 -1 +1  0
+1  0 +1  0
  0 -1 -1 +1
-1 +1 -1 +1
+1 -1  0 +1
  0 +1  0 +1
-1  0 +1 +1
+1 +1 +1 +1

```

|               |       |       |       |       |       |       |       |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| actual Model: | x1    | x2    | x3    | x4    | x11   | x22   | x33   |
| fitted Coefs: | 1.00  | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  |
| actual Model: | x2    | x3    | x4    | x11   | x22   | x33   | x44   |
| fitted Coefs: | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  |
| actual Model: | x3    | x4    | x11   | x22   | x33   | x44   | x12   |
| fitted Coefs: | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  |
| actual Model: | x4    | x11   | x22   | x33   | x44   | x12   | x13   |
| fitted Coefs: | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 |
| actual Model: | x11   | x22   | x33   | x44   | x12   | x13   | x14   |
| fitted Coefs: | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 |
| actual Model: | x22   | x33   | x44   | x12   | x13   | x14   | x23   |
| fitted Coefs: | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 |
| actual Model: | x33   | x44   | x12   | x13   | x14   | x23   | x24   |
| fitted Coefs: | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 |
| actual Model: | x44   | x12   | x13   | x14   | x23   | x24   | x34   |
| fitted Coefs: | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 |
| actual Model: | x12   | x13   | x14   | x23   | x24   | x34   | x123  |
| fitted Coefs: | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 |
| actual Model: | x13   | x14   | x23   | x24   | x34   | x123  | x124  |
| fitted Coefs: | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 |
| actual Model: | x14   | x23   | x24   | x34   | x123  | x124  | x134  |
| fitted Coefs: | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 |
| actual Model: | x23   | x24   | x34   | x123  | x124  | x134  | x234  |
| fitted Coefs: | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | .00   | 19.00 |

14 Points ( very sixth point in the population)

```

-1 -1 -1 -1
-1 +1 -1 -1
-1 0 0 -1
-1 -1 +1 -1
-1 +1 +1 -1
-1 0 -1 0
-1 -1 0 0
-1 +1 0 0
-1 0 +1 0
-1 -1 -1 +1
-1 +1 -1 +1
-1 0 0 +1
-1 -1 +1 +1
-1 +1 +1 +1

```

|               |              |              |              |              |             |              |             |
|---------------|--------------|--------------|--------------|--------------|-------------|--------------|-------------|
| actual Model: | x1           | x2           | x3           | x4           | x11         | x22          | x33         |
| fitted Coefs: | <u>-2.00</u> | 2.00         | 3.00         | 4.00         | <u>2.00</u> | 6.00         | 7.00        |
| actual Model: | x2           | x3           | x4           | x11          | x22         | x33          | x44         |
| fitted Coefs: | 2.00         | 3.00         | 4.00         | 5.00         | 6.00        | 7.00         | 8.00        |
| actual Model: | x3           | x4           | x11          | x22          | x33         | x44          | x12         |
| fitted Coefs: | 3.00         | 4.00         | 5.00         | 6.00         | 7.00        | 8.00         | 9.00        |
| actual Model: | x4           | x11          | x22          | x33          | x44         | x12          | x13         |
| fitted Coefs: | 4.00         | 5.00         | 6.00         | 7.00         | 8.00        | 9.00         | 10.00       |
| actual Model: | x11          | x22          | x33          | x44          | x12         | x13          | x14         |
| fitted Coefs: | 5.00         | 6.00         | 7.00         | 8.00         | 9.00        | 10.00        | 11.00       |
| actual Model: | x22          | x33          | x44          | x12          | x13         | x14          | x23         |
| fitted Coefs: | 6.00         | 7.00         | 8.00         | 9.00         | 10.00       | 11.00        | 12.00       |
| actual Model: | x33          | x44          | x12          | x13          | x14         | x23          | x24         |
| fitted Coefs: | 7.00         | 8.00         | 9.00         | 10.00        | 11.00       | 12.00        | 13.00       |
| actual Model: | x44          | x12          | x13          | x14          | x23         | x24          | x34         |
| fitted Coefs: | 8.00         | 9.00         | 10.00        | 11.00        | 12.00       | 13.00        | 14.00       |
| actual Model: | x12          | x13          | x14          | x23          | x24         | x34          | x123        |
| fitted Coefs: | 9.00         | 10.00        | 11.00        | <u>-1.50</u> | 13.00       | 14.00        | <u>1.50</u> |
| actual Model: | x13          | x14          | x23          | x24          | x34         | x123         | x124        |
| fitted Coefs: | 10.00        | 11.00        | <u>-1.50</u> | <u>-1.50</u> | 14.00       | <u>1.50</u>  | <u>1.50</u> |
| actual Model: | x14          | x23          | x24          | x34          | x123        | x124         | x134        |
| fitted Coefs: | 11.00        | <u>-1.50</u> | <u>-1.50</u> | <u>-1.50</u> | 1.50        | 1.50         | 1.50        |
| actual Model: | x23          | x24          | x34          | x123         | x124        | x134         | x234        |
| fitted Coefs: | <u>-1.50</u> | <u>-1.50</u> | <u>-1.50</u> | 1.50         | 1.50        | 1.50         | 18.00       |
| actual Model: | x24          | x34          | x123         | x124         | x134        | x234         | x1234       |
| fitted Coefs: | <u>-1.50</u> | <u>-1.50</u> | 15.00        | <u>1.50</u>  | 1.50        | <u>-.50</u>  | <u>.50</u>  |
| actual Model: | x24          | x34          | x123         | x124         | x134        | x234         | x1234       |
| fitted Coefs: | <u>-8.00</u> | <u>-1.50</u> | 15.00        | 8.00         | 1.50        | <u>-.50</u>  | <u>.50</u>  |
| actual Model: | x24          | x34          | x123         | x124         | x134        | x234         | x1234       |
| fitted Coefs: | <u>-8.00</u> | <u>-8.50</u> | 15.00        | 8.00         | 8.50        | <u>-.50</u>  | <u>.50</u>  |
| actual Model: | x24          | x34          | x123         | x124         | x134        | x234         | x1234       |
| fitted Coefs: | <u>-8.00</u> | <u>-8.50</u> | .00          | 8.00         | 8.50        | <u>-.50</u>  | <u>.50</u>  |
| actual Model: | x24          | x34          | x123         | x124         | x134        | x234         | x1234       |
| fitted Coefs: | .00          | <u>-8.50</u> | .00          | .00          | 8.50        | <u>-.50</u>  | <u>.50</u>  |
| actual Model: | x24          | x34          | x123         | x124         | x134        | x234         | x1234       |
| fitted Coefs: | .00          | .00          | .00          | .00          | .00         | <u>-.50</u>  | <u>.50</u>  |
| actual Model: | x24          | x34          | x123         | x124         | x134        | x234         | x1234       |
| fitted Coefs: | .00          | .00          | .00          | .00          | .00         | <u>-9.50</u> | <u>9.50</u> |

12 Points (every seventh point in the population)

```

-1  -1  -1  -1
 0  +1  -1  -1
+1   0   0  -1
-1   0  +1  -1
 0  -1  -1   0
+1  +1  -1   0
-1  +1   0   0
 0   0  +1   0
+1  -1  -1  +1
-1  -1   0  +1
 0  +1   0  +1
+1   0  +1  +1

```

|               |       |       |       |       |       |       |       |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| actual Model: | x1    | x2    | x3    | x4    | x11   | x22   | x33   |
| fitted Coefs: | 1.00  | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  |
| actual Model: | x2    | x3    | x4    | x11   | x22   | x33   | x44   |
| fitted Coefs: | 2.00  | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  |
| actual Model: | x3    | x4    | x11   | x22   | x33   | x44   | x12   |
| fitted Coefs: | 3.00  | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  |
| actual Model: | x4    | x11   | x22   | x33   | x44   | x12   | x13   |
| fitted Coefs: | 4.00  | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 |
| actual Model: | x11   | x22   | x33   | x44   | x12   | x13   | x14   |
| fitted Coefs: | 5.00  | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 |
| actual Model: | x22   | x33   | x44   | x12   | x13   | x14   | x23   |
| fitted Coefs: | 6.00  | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 |
| actual Model: | x33   | x44   | x12   | x13   | x14   | x23   | x24   |
| fitted Coefs: | 7.00  | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 |
| actual Model: | x44   | x12   | x13   | x14   | x23   | x24   | x34   |
| fitted Coefs: | 8.00  | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 |
| actual Model: | x12   | x13   | x14   | x23   | x24   | x34   | x123  |
| fitted Coefs: | 9.00  | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 |
| actual Model: | x13   | x14   | x23   | x24   | x34   | x123  | x124  |
| fitted Coefs: | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 |
| actual Model: | x14   | x23   | x24   | x34   | x123  | x124  | x134  |
| fitted Coefs: | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 |
| actual Model: | x23   | x24   | x34   | x123  | x124  | x134  | x234  |
| fitted Coefs: | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | 16.00 | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | 17.00 | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | 18.00 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | .00   | 19.00 |

11 Points (every eighth point in the Population)

```

-1  -1  -1  -1
+1  +1  -1  -1
  0  +1   0  -1
-1  +1  +1  -1
+1   0  -1   0
  0   0   0   0
-1   0  +1   0
+1  -1  -1  +1
  0  -1   0  +1
-1  -1  +1  +1
+1  +1  +1  +1

```

|               |       |       |       |       |       |       |       |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| actual Model: | x1    | x2    | x3    | x4    | x11   | x22   | x33   |
| fitted Coefs: | 1.50  | 1.50  | 3.50  | 3.50  | 6.00  | 6.00  | 6.00  |
| actual Model: | x2    | x3    | x4    | x11   | x22   | x33   | x44   |
| fitted Coefs: | 2.00  | 3.00  | 4.00  | 6.00  | 7.00  | 6.00  | 7.00  |
| actual Model: | x3    | x4    | x11   | x22   | x33   | x44   | x12   |
| fitted Coefs: | 3.00  | 4.00  | 6.00  | 7.00  | 6.00  | 7.00  | 9.00  |
| actual Model: | x4    | x11   | x22   | x33   | x44   | x12   | x13   |
| fitted Coefs: | 4.00  | 6.00  | 7.00  | 6.00  | 7.00  | 9.00  | 10.00 |
| actual Model: | x11   | x22   | x33   | x44   | x12   | x13   | x14   |
| fitted Coefs: | 6.57  | 7.00  | 6.57  | 7.00  | 7.86  | 11.14 | 9.86  |
| actual Model: | x22   | x33   | x44   | x12   | x13   | x14   | x23   |
| fitted Coefs: | 7.00  | 8.00  | 7.00  | 8.00  | 11.00 | 11.00 | 11.00 |
| actual Model: | x33   | x44   | x12   | x13   | x14   | x23   | x24   |
| fitted Coefs: | 8.30  | 7.30  | 8.40  | 11.30 | 11.20 | 11.20 | 12.30 |
| actual Model: | x44   | x12   | x13   | x14   | x23   | x24   | x34   |
| fitted Coefs: | 8.67  | 11.17 | 10.00 | 11.17 | 11.17 | 13.67 | 11.17 |
| actual Model: | x12   | x13   | x14   | x23   | x24   | x34   | x123  |
| fitted Coefs: | 11.50 | 10.00 | 11.50 | 11.50 | 13.00 | 11.50 | 15.00 |
| actual Model: | x13   | x14   | x23   | x24   | x34   | x123  | x124  |
| fitted Coefs: | 10.00 | 11.50 | 11.50 | 13.00 | 14.00 | 15.00 | 16.00 |
| actual Model: | x14   | x23   | x24   | x34   | x123  | x124  | x134  |
| fitted Coefs: | 11.50 | 11.50 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 |
| actual Model: | x23   | x24   | x34   | x123  | x124  | x134  | x234  |
| fitted Coefs: | 12.00 | 13.00 | 14.00 | 15.50 | 15.50 | 17.50 | 17.50 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | 13.00 | 14.00 | 15.50 | 15.50 | 17.50 | 17.50 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | 14.00 | 15.50 | 15.50 | 17.50 | 17.50 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | 15.50 | 15.50 | 17.50 | 17.50 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | 4.25  | 11.75 | 21.25 | 13.75 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .25   | -.25  | 17.25 | 17.75 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | 4.50  | -4.50 | 4.50  | 13.50 | 19.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .00   | .00   | .00   | .00   | .00   | 19.00 |

9 Points (every ninth point in the population)

```

-1 -1 -1 -1
-1 -1  0 -1
-1 -1 +1 -1
-1 -1 -1  0
-1 -1  0  0
-1 -1 +1  0
-1 -1 -1 +1
-1 -1  0 +1
-1 -1 +1 +1

```

|               |       |       |       |       |       |       |       |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| actual Model: | x1    | x2    | x3    | x4    | x11   | x22   | x33   |
| fitted Coefs: | -2.00 | -2.00 | 3.00  | 4.00  | 2.00  | 2.00  | 7.00  |
| actual Model: | x2    | x3    | x4    | x11   | x22   | x33   | x44   |
| fitted Coefs: | -3.00 | 3.00  | 4.00  | 3.00  | 3.00  | 7.00  | 8.00  |
| actual Model: | x3    | x4    | x11   | x22   | x33   | x44   | x12   |
| fitted Coefs: | 3.00  | 4.00  | 6.67  | 6.67  | 7.00  | 8.00  | 6.67  |
| actual Model: | x4    | x11   | x22   | x33   | x44   | x12   | x13   |
| fitted Coefs: | 4.00  | 6.67  | 6.67  | 7.00  | 8.00  | 6.67  | 10.00 |
| actual Model: | x11   | x22   | x33   | x44   | x12   | x13   | x14   |
| fitted Coefs: | 6.67  | 6.67  | 7.00  | 8.00  | 6.67  | 10.00 | 11.00 |
| actual Model: | x22   | x33   | x44   | x12   | x13   | x14   | x23   |
| fitted Coefs: | 7.50  | 7.00  | 8.00  | 7.50  | 11.00 | 11.00 | 11.00 |
| actual Model: | x33   | x44   | x12   | x13   | x14   | x23   | x24   |
| fitted Coefs: | 7.00  | 8.00  | 9.00  | 11.00 | 12.00 | 11.00 | 12.00 |
| actual Model: | x44   | x12   | x13   | x14   | x23   | x24   | x34   |
| fitted Coefs: | 8.00  | 9.00  | 11.00 | 12.00 | 11.00 | 12.00 | 14.00 |
| actual Model: | x12   | x13   | x14   | x23   | x24   | x34   | x123  |
| fitted Coefs: | 9.00  | 2.33  | 12.00 | 2.33  | 12.00 | 14.00 | -2.33 |
| actual Model: | x13   | x14   | x23   | x24   | x34   | x123  | x124  |
| fitted Coefs: | 2.33  | 2.67  | 2.33  | 2.67  | 14.00 | -2.33 | -2.67 |
| actual Model: | x14   | x23   | x24   | x34   | x123  | x124  | x134  |
| fitted Coefs: | 2.67  | -1.50 | 2.67  | -1.50 | 1.50  | -2.67 | 1.50  |
| actual Model: | x23   | x24   | x34   | x123  | x124  | x134  | x234  |
| fitted Coefs: | -1.50 | -1.50 | -7.00 | 1.50  | 1.50  | 7.00  | 7.00  |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | -1.50 | -.50  | 15.00 | 1.50  | .50   | .50   | -.50  |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | -8.00 | -.50  | 15.00 | 8.00  | .50   | .50   | -.50  |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | -8.00 | -4.00 | 15.00 | 8.00  | 4.00  | 4.00  | -4.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | -8.00 | -4.00 | .00   | 8.00  | 4.00  | 4.00  | -4.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | -4.00 | .00   | .00   | 4.00  | 4.00  | -4.00 |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | .25   | .00   | .00   | -.25  | -.25  | .25   |
| actual Model: | x24   | x34   | x123  | x124  | x134  | x234  | x1234 |
| fitted Coefs: | .00   | 4.75  | .00   | .00   | -4.75 | -4.75 | 4.75  |